On Mopping:
A Mathematical Model for Mopping a Dirty Floor

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Abstract

Several times in my life I have been told that mopping the floor is not a valid method of cleaning because it “just spreads the filth around”. This is clearly false. As anyone who has mopped a floor can testify, the initially clean water in the mop bucket is saturated with dirt by the end of the cleaning session. This dirt started on the floor and ended in the bucket, so clearly the floor has less dirt on it after mopping. Even after this simple observation, some critics of mopping remain unconvinced. The notion that a centuries-old janitorial technique serves no purpose is as insulting to those of the profession as it is false. Decriers of the method may be motivated by a desire to appear worldly and experienced to their peers, or perhaps they may simply trying to avoid their mopping duties. This paper does not attempt to provide insight into the motivations of such naysayers, but merely aims quell disbelief in the effectiveness of mopping by use of a mathematical model.

Model

Cleaning method

The following models the situation where an initially clean mop and bucket of water are used to clean a floor. First, the mop is wetted with the maximum amount of water it can hold. Next, the mop is dragged across the floor, collecting dirt. Dirt continues to collect in the water on the mop until the maximum saturation of the mop water is reached. The mop is then dipped into the water in the bucket, depositing all of its suspended dirt and increasing the concentration of dirt in the bucket of water. The mop is then removed from the water again, this time however, the water on the mop is not entirely clean. The mop water has the same concentration of dirt as the bucket water, and therefore can collect less net dirt from the floor before being saturated again. This continues until the water in the bucket is saturated with dirt and the mop removed from the water is also saturated. Because the water on the mop already holds the maximum possible amount of dirt, it can no longer remove dirt from the floor. At this point the water and mop must be cleaned. This process is repeated until a sufficient amount of dirt has been collected from the floor.

Assumptions

1. The mop and bucket are initially clean.
2. There is a maximum concentration of dirt that water can hold, by dragging the mop across the floor, this concentration is reached in the mop.
3. The amount of water deposited on the floor by the mop is negligible.
4. All solutions are well-mixed.

Let $Q(t)$ represent the concentration of dirt in the water bucket at any given time. Note that the units of $Q(t)$ are grams per liter, and $Q(0) = 0$ as the water is initially clean. The volume of water in the bucket is V_B liters. Each time the dirty mop is returned to the bucket, some amount of dirt, dD, is deposited in the bucket water. This changes the concentration of the bucket water by the following equation.

$$\frac{dQ}{dt} = \frac{dD}{V_T}$$

The amount of dirt deposited, dD, depends on the maximum amount of water that the mop can hold, V_M. The water on the mop starts with an initial concentration of dirt equal to that of the water in the bucket. Dirt is then added to the water on the mop until the water is saturated. If Q_{max} represents the maximum concentration of dirt in water, the following equation shows that the final saturation of the mop comes from both the initial dirt on the mop, $Q(t)$, and the dirt picked up from the floor, dD.

$$Q_{max} = Q(t) + \frac{dD}{V_M}$$

Rearranging

$$dD = Q_{max}V_M - Q(t)V_M$$

Substituting this value into the first equation

$$\frac{dQ}{dt} = \frac{V_M}{V_T}Q_{max} - \frac{V_M}{V_T}Q(t)$$

$$\frac{dQ}{dt} + \frac{V_M}{V_T}Q(t) = \frac{V_M}{V_T}Q_{max}$$

A first-order linear differential equation which can be solved using the integrating factor

$$\mu = e^{\frac{V_M}{V_T}t}$$

$$\frac{dQ}{dt}e^{\frac{V_M}{V_T}t} + \frac{V_M}{V_T}Q(t)e^{\frac{V_M}{V_T}t} = \frac{V_M}{V_T}Q_{max}e^{\frac{V_M}{V_T}t}$$

$$\frac{d}{dt}\left(Q(t)e^{\frac{V_M}{V_T}t}\right) = \frac{V_M}{V_T}Q_{max}e^{\frac{V_M}{V_T}t}$$

$$d\left(Q(t)e^{\frac{V_M}{V_T}t}\right) = \frac{V_M}{V_T}Q_{max}e^{\frac{V_M}{V_T}t}dt$$

$$\int d\left(Q(t)e^{\frac{V_M}{V_T}t}\right) = \int \frac{V_M}{V_T}Q_{max}e^{\frac{V_M}{V_T}t}dt$$

$$Q(t)e^{\frac{V_M}{V_T}t} = Q_{max}e^{\frac{V_M}{V_T}t} + C$$

$$Q(t) = Q_{max} + Ce^{-\frac{V_M}{V_T}t}$$

Using the initial condition

$$Q(0) = 0$$

$$0 = Q_{max} + C$$

$$C = -Q_{max}$$

Giving the final function

$$Q(t) = Q_{max} - Q_{max}e^{-\frac{V_M}{V_T}t}$$

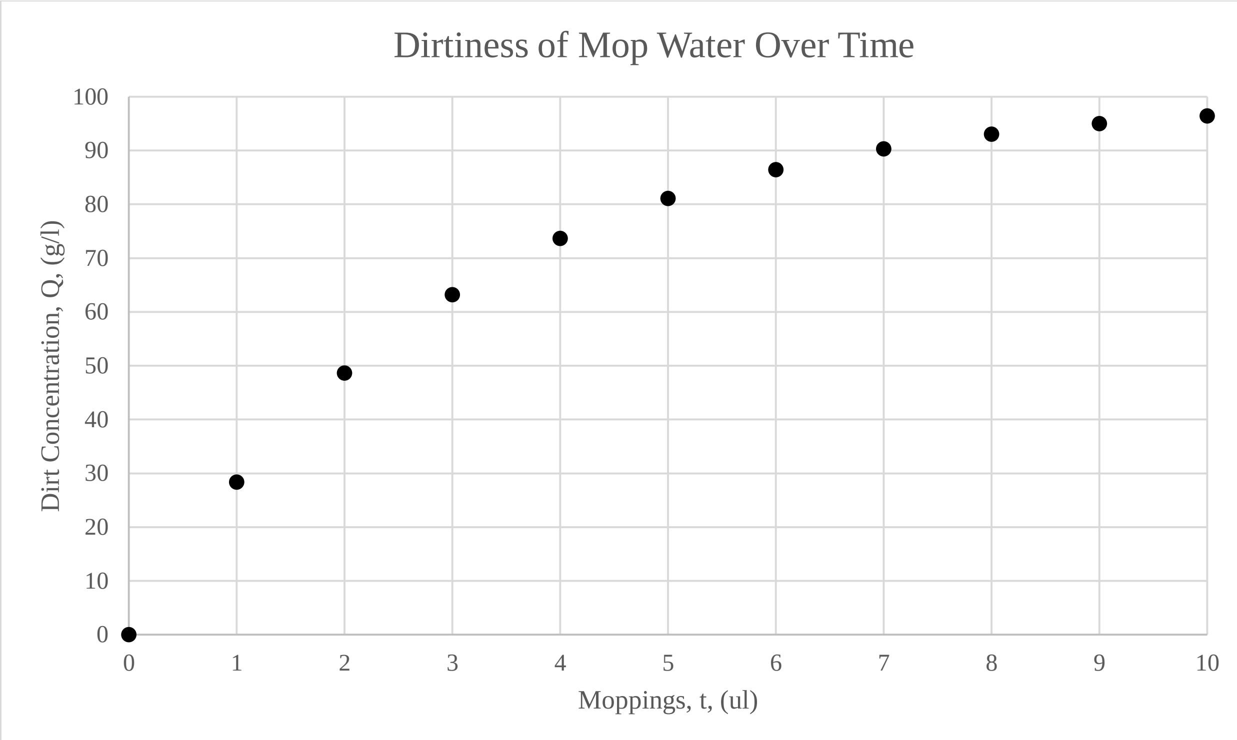

Figure 1. Plot of one solution with Q_{max} *as 100 g/l.*

Conclusions

This model shows, as expected, that the marginal utility of mopping the floor decreases as the mop water gets increasingly dirty. However, the notion that mopping the floor just "spreads the dirt around" is clearly false as each mopping moves dD grams of dirt from the floor into the bucket of water. The target audience of this paper is those who, perhaps in an attempt to seem learned on the topic, decry mopping as an ineffective method for cleaning floors, yet poses a basic understanding of ordinary differential equations. The techniques presented here could be used in further research to disprove the idea that taking a bath is merely "soaking in one's own filth".